LAWS OF MOTION & FRICTION

Dynamics :

Motion or state of the body under the given force

Newton's Law of Motion

First Law: Every body tries to continue in a state of rest or in a uniform motion unless the force is impressed upon it. If the body or system has no net force i.e., in equilibrium, then body is either in a sate of rest or in a uniform motion (Motion with constant speed along straightline).

Inferences of First Law

(a) Every body has tendency not to change the state of rest or uniform motion, measure of this tendency called intertia so first law often called law of inertia. Inertia measures the tendency not to change the state either from state of rest or in uniform motion.

(b) Definition of force also comes from frist law and it is defined as the cause that changes the state either from state of rest or uniform motion.

(c) Frame of references are of two kinds

(i) Inertial frame of reference : The frame of reference which is either at rest or moving in uniform motion. Newton's laws are valid only in inertial frame of reference. So it is also called Newtonian frame of reference.

(ii) Non-inertial frame : The frame of reference which is in accelerated motion. Newton's laws are not valid in non-intertial frame.

(d) There are only two natural states of body (i) At rest and (ii) in uniform motion.

Newton's Second Law of Motion:

The time rate of change of linear momentum of a body is directly proportional to force experienced and direction of force is in the direction of change in linear momentum.

Linear Momentum of a Particle is the quantity of motion and mathematically defined as the product of mass and velocity, $\vec{p} = m\vec{v}$.

From Newton's 2nd Law

$$\frac{\overrightarrow{dp}}{dt} \propto \vec{F} \Rightarrow \frac{\overrightarrow{dp}}{dt} = \kappa\vec{F} \quad \text{from definition of one newton, } \kappa = 1$$

1 newton is the force experienced by a body when the rate of change of momentum is unity.

$$\therefore \frac{\overrightarrow{dp}}{dt} = \vec{F} \quad \text{...(i) and} \quad \frac{d(m\vec{v})}{dt} = \vec{F} \Rightarrow \vec{F} = m\vec{a} \quad \text{...(ii)}$$

It $\vec{a} = 0$ then $\vec{F} = 0$, i.e., no force means unaccelerated motion.

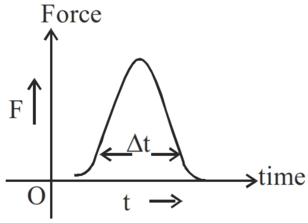
$$d\vec{p} = \vec{F}\cdot dt \Rightarrow \int_{\vec{p}_i}^{\vec{p}_f} \overrightarrow{dp} = \int_0^{\Delta t} \vec{F}\cdot dt \Rightarrow \vec{p}_f - \vec{p}_i = \vec{F}.\Delta t$$

$\vec{F}$ is the force that acts for relatively small time with very large value which is called impulsive force.

Where $\vec{F}.\Delta t$ is the impulse.

Change in linear momentum is equal to the impulse imparted.

Impulse is sudden jerk which causes to change the linear momentum.

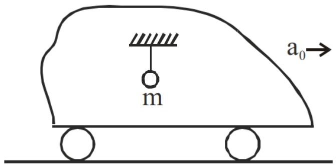


Area under the force vs time curve for impulse imparted gives the change in linear momentum.

Pseudoforce : A force on a body as a consequence of non inertial frame of reference it is equal to the product of mass of the body and acceleration of the frame with respect to an a accelerated car and it is directed oppsite to direction of acceleration of frame of reference.

Equation of motion for a simple pendulum w.r.t. to ground $\vec{T}+\vec{w}=m\vec{a}_0$ ...(i)

Equation of Motion for simple pendulum with respect to a car.



$\vec{T}+\vec{w}=0$ ...(ii)

but it is incorrect

Since frame of reference is accelerated

To write correct equation for (ii) we use (i) $\vec{T}+\vec{w}=m\vec{a}_0$

Now equation of mass m w.r.t to car be written correctly as $\vec{T}+\vec{w}-m\vec{a}_0=0$ ....(iii)

This is the correct equation of motion with respect to accelerated frame. Where $-m\vec{a}_0$ called pseudo force

$\vec{a}_0$ is the acceleration of frame.

Kind of forces:

(i) Normal reaction: Force on a body due to one another perpendicular to the line of contact.

(ii) Weight : Force by which a body is being attracted towards earth.

(iii) Tension :An elastic force that comes within string to oppose the tendency of deformation, it acts away from system along the string. For massless string tension at every point is same in magnitude.

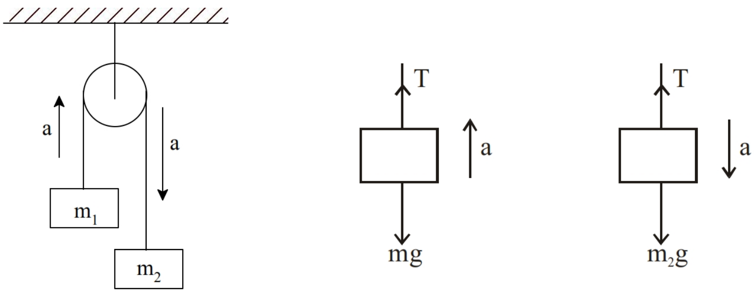
Newton's Third Law of Motion: For a system there are equal and opposite forces on a pair of body due to one another. The one force called action while another called reaction. Total internal forces on a system is always zero.

Finding acceleration through free body diagram

Motion of connected bodies

Illustrations 1 :

In Atwood machine find the acceleration of m_1 and m_2 where pulley and string are massless and pulley is smooth. $a=$ acceleration of m_1 or m_2.



Solution

Equation of motion for m_1

$T - m_1 = m_1 a$ ...(1)

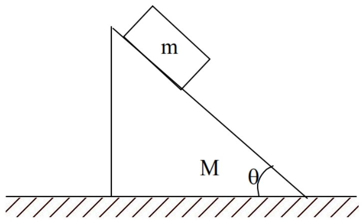
Equation of mass m_2

$m_2 g - T = m_2 a$ ...(2)

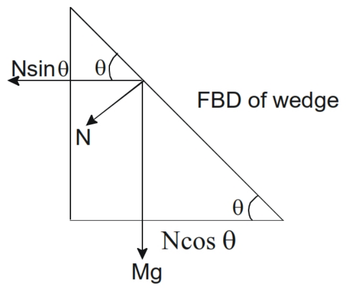
From (1) an (2) we have $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2}$

Illustrations : 2

All the surfaces are smooth for a system of wedge and block. Acceleration of wedge after system is released. Then acceleration of wedge is given by as follows,



Equation of motion for wedge M



$N \sin\theta = Ma$ ...(i)

Let a_r be the acceleration of block w.r.t wedge

Equation for mass m

$mg\sin\theta + mg\cos\theta = m.a_r$ ...(ii)

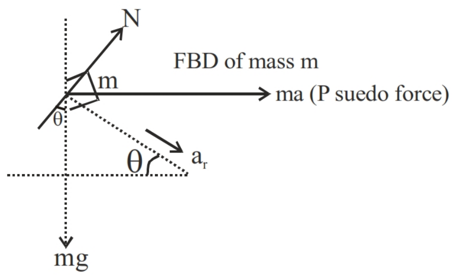
and $N + ma\sin\theta = mg\cos\theta$ ...(iii)

From equation (1) and (iii)

$(Mg\cos\theta - ma\sin\theta)\cdot\sin\theta = Ma$

$\Rightarrow \quad a(M + m\sin^2\theta) = Mg\cos\theta\cdot\sin\theta$

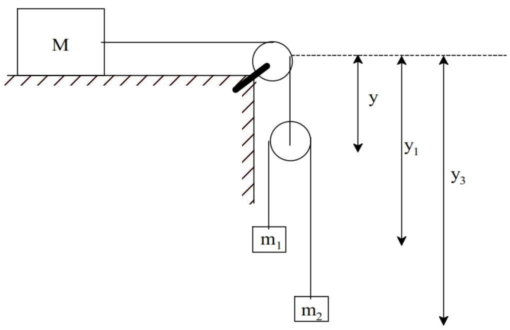
$a = \dfrac{Mg\cos\theta\cdot\sin\theta}{M + m\sin^2\theta}$



EQUATION OF CONSTRAINTS

Equation involving the acceleration of connecting bodies is the equation of constraints

Illustrations 3 :



L = length of spring which is on the movable pulley.

$(y_2 - y) + (y_1 - y) + \pi R = \ell \quad \ldots(1)$

R = radius of pulley

Differentiating equation (1) w.r.t time twice

$$\frac{d^2y_1}{dt^2} + \frac{d^2y_2}{dt^2} = 2\frac{d^2y_1}{dt^2}$$

$$\Rightarrow \qquad a_1 + a_2 = 2 \cdot a \Rightarrow a = \frac{a_1 + a_2}{2}$$

a = acceleration of block of mass M

a_1 = acceleration of block of mass m_1

a_2 = acceleration of block of mass m_2

Illustrations 4 :

a_1 = acceleration of m_1

a_2 = acceleration of m_1

L = length of spring

$= \ell - x_1 + y_1 + \ell$

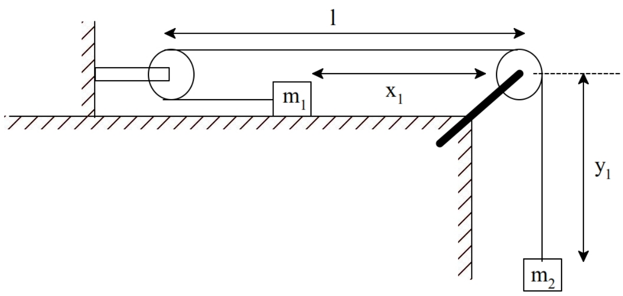
$= 2\ell - x_1 + y_1$

Differentiating w.r.t. time time-twice

$$\frac{d^2x_1}{dt^2} = \frac{d^2y_1}{dt^2} \Rightarrow a_1 = a_2$$

a_1 = acceleration of m_1

a_2 = acceleration of m_1



Note :

Weight shown in the spring is the tension in the spring and weight shown in the weighing machine is the normal reaction acts on the weighing machine.

FRICTION

When two bodies are either having a tendency of relative motion or a relative motion, a force comes into play along the line of motion to oppose the tendency of relative motion or relative motion. Such phenomenon is called friction and force is called frictional force.

Static friction : Force of friction to oppose the tendency of relative motion called static frictional force which is directly proportional to normal reaction and independent of the area of contact.

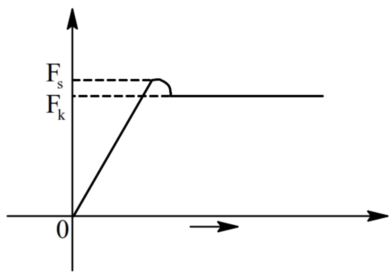
i.e., $F_s \propto N$ or $F_s = \mu_s N$

where μ_s is the co-efficient of static friction.

Kinetic friction:

Force of kinetic friction is force of friction to oppose the relative motion and which is given by F_K and F_K is directly proportional to normal reaction and also independent of the area of contact.

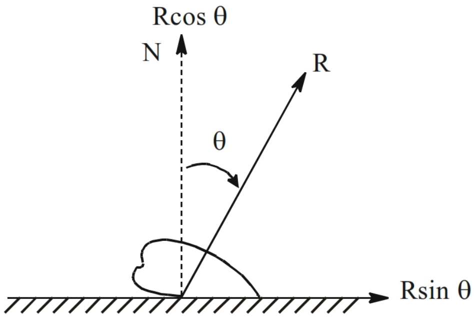
i.e., $F_K \propto N$ or $F_K = \mu_K . N$



F_s (static friction) is a self-adjusting force of friction, maximum value of it is the limiting friction.

$F_s \leq \mu_s N$. Direction of force of friction is in opposite to direction of either tendency of relative motion or relative motion.

Angle of friction



When two rough bodies are is contact then there is a force of equal magnitude but opposite in direction are another called contact force (R) Horizontal component of R provides the force of friction while its normal component is the normal reaction ; R is contact force on a body due to surface.

$$F_s = R\sin\theta \text{ and } N = R\cos\theta$$

$$\therefore \frac{F_s}{N} = \tan\theta \Rightarrow \tan\theta = \mu_s \text{ or } \theta = \tan^{-1}(\mu_s) < 45^\circ \text{ As } \mu_s < 1$$

θ is the angle of friction

Angle of Repose :

The maximum angle of inclination for which the block on rough inclined plane is on the verge of sliding called angle of repose.

Here $mg \sin\theta = \mu_s N$

$= \mu_s \, mg \cos\theta$

$\Rightarrow \tan\theta = \mu_s$ or $\theta = \tan^{-1}(\mu_s)$; θ is the angle of repose.